Set Associative Mapping | Practice Problems

Set Associative Mapping-

In set associative mapping,

- A particular block of main memory can be mapped to one particular cache set only.
- Block 'j' of main memory will map to set number (j mod number of sets in cache) of the cache.
- A replacement algorithm is needed if the cache is full.

In this article, we will discuss practice problems based on set associative mapping.

PRACTICE PROBLEMS BASED ON SET ASSOCIATIVE MAPPING-

Problem-01:

Consider a 2-way set associative mapped cache of size 16 KB with block size 256 bytes. The size of main memory is 128 KB. Find-

1. Number of bits in tag
2. Tag directory size

Solution-

Given-

- Set size = 2
- Cache memory size = 16 KB
- Block size = Frame size = Line size = 256 bytes
- Main memory size = 128 KB

We consider that the memory is byte addressable.

Number of Bits in Physical Address-

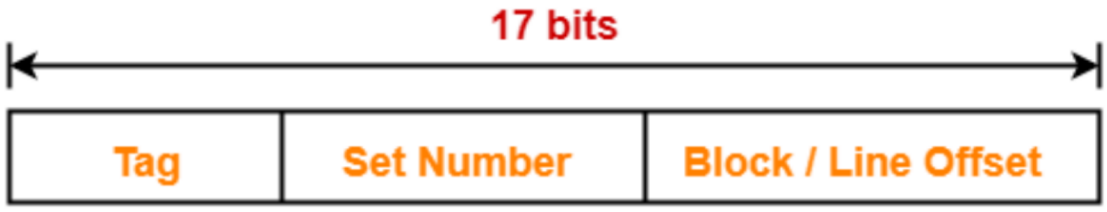
We have,

Size of main memory

= 128 KB

= 2^{17} bytes

Thus, Number of bits in physical address = 17 bits



Number of Bits in Block Offset-

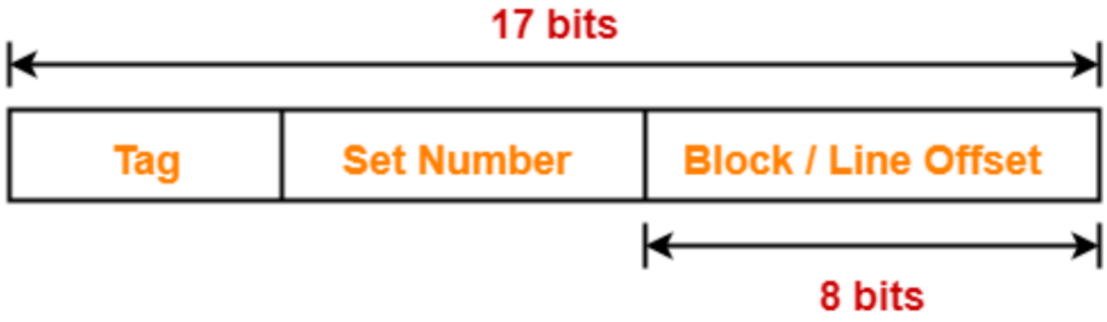
We have,

Block size

= 256 bytes

= 2^{8} bytes

Thus, Number of bits in block offset = 8 bits



Number of Lines in Cache-

Total number of lines in cache

= Cache size / Line size

= 16 KB / 256 bytes

= 2^{14} bytes / 2^{8} bytes

= 64 lines

Thus, Number of lines in cache = 64 lines

Number of Sets in Cache-

Total number of sets in cache

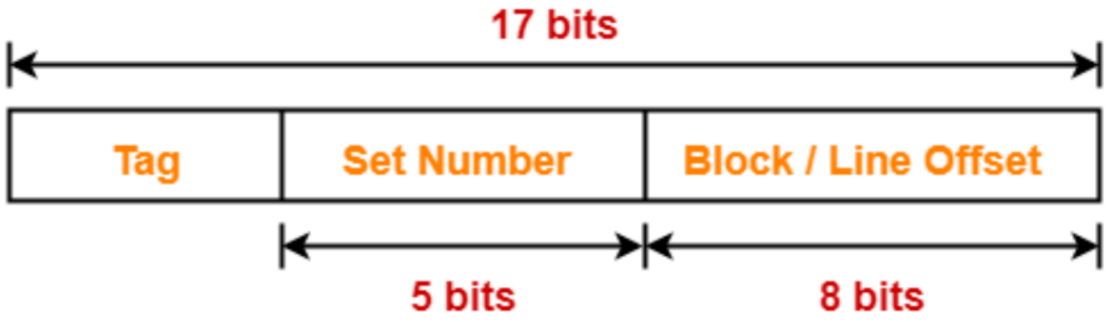
= Total number of lines in cache / Set size

= 64 / 2

= 32 sets

= 2^{5} sets

Thus, Number of bits in set number = 5 bits



Number of Bits in Tag-

Number of bits in tag

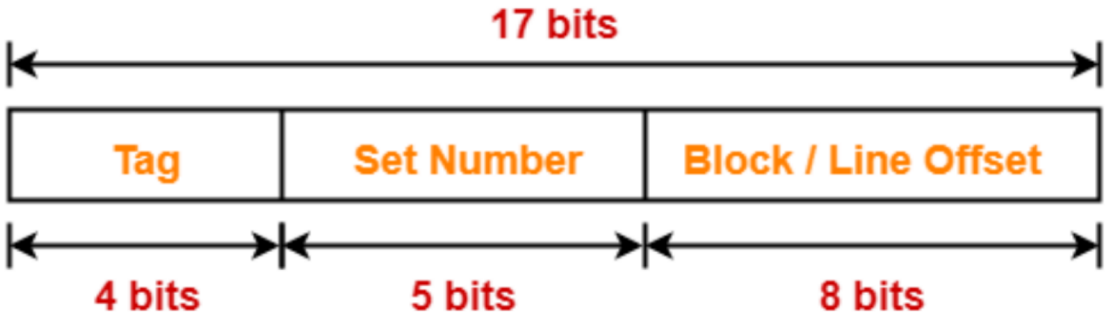
= Number of bits in physical address – (Number of bits in set number + Number of bits in block offset)

= 17 bits – (5 bits + 8 bits)

= 17 bits – 13 bits

= 4 bits

Thus, Number of bits in tag = 4 bits



Tag Directory Size-

Tag directory size

= Number of tags x Tag size

= Number of lines in cache x Number of bits in tag

= 64 x 4 bits

= 256 bits

= 32 bytes

Thus, size of tag directory = 32 bytes

Problem-02:

Consider a 8-way set associative mapped cache of size 512 KB with block size 1 KB. There are 7 bits in the tag. Find-

1. Size of main memory
2. Tag directory size

Solution-

Given-

- Set size = 8
- Cache memory size = 512 KB
- Block size = Frame size = Line size = 1 KB

- Number of bits in tag = 7 bits

We consider that the memory is byte addressable.

Number of Bits in Block Offset-

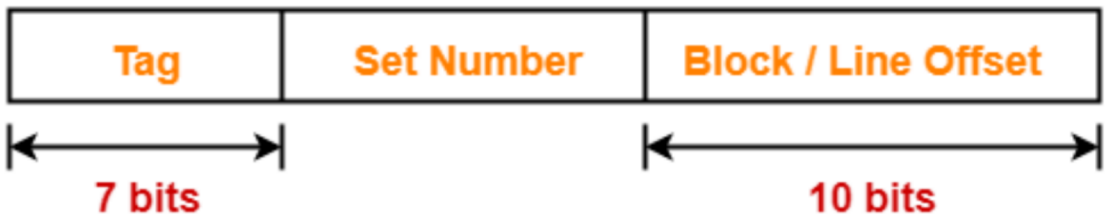
We have,

Block size

= 1 KB

= 2^{10} bytes

Thus, Number of bits in block offset = 10 bits



Number of Lines in Cache-

Total number of lines in cache

= Cache size / Line size

= 512 KB / 1 KB

= 512 lines

Thus, Number of lines in cache = 512 lines

Number of Sets in Cache-

Total number of sets in cache

= Total number of lines in cache / Set size

= 512 / 8

= 64 sets

= 2^6 sets

Thus, Number of bits in set number = 6 bits

Number of Bits in Physical Address-

Number of bits in physical address

= Number of bits in tag + Number of bits in set number + Number of bits in block offset

= 7 bits + 6 bits + 10 bits

= 23 bits

Thus, Number of bits in physical address = 23 bits

Size of Main Memory-

We have,

Number of bits in physical address = 23 bits

Thus, Size of main memory

= 2^{23} bytes

= 8 MB

Tag Directory Size-

Tag directory size

= Number of tags x Tag size

= Number of lines in cache x Number of bits in tag

= 512 x 7 bits

= 3584 bits

= 448 bytes

Thus, size of tag directory = 448 bytes

Problem-03:

Consider a 4-way set associative mapped cache with block size 4 KB. The size of main memory is 16 GB and there are 10 bits in the tag. Find-

1. Size of cache memory
2. Tag directory size

Solution-

Given-

- Set size = 4
- Block size = Frame size = Line size = 4 KB
- Main memory size = 16 GB
- Number of bits in tag = 10 bits

We consider that the memory is byte addressable.

Number of Bits in Physical Address-

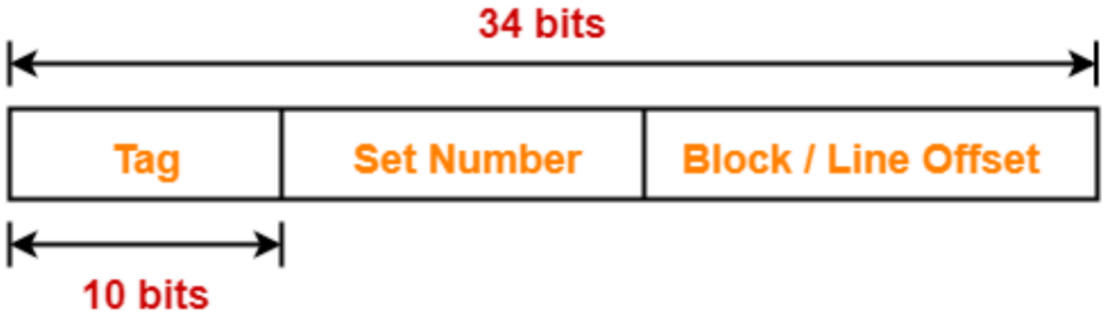
We have,

Size of main memory

= 16 GB

= 2^{34} bytes

Thus, Number of bits in physical address = 34 bits



Number of Bits in Block Offset-

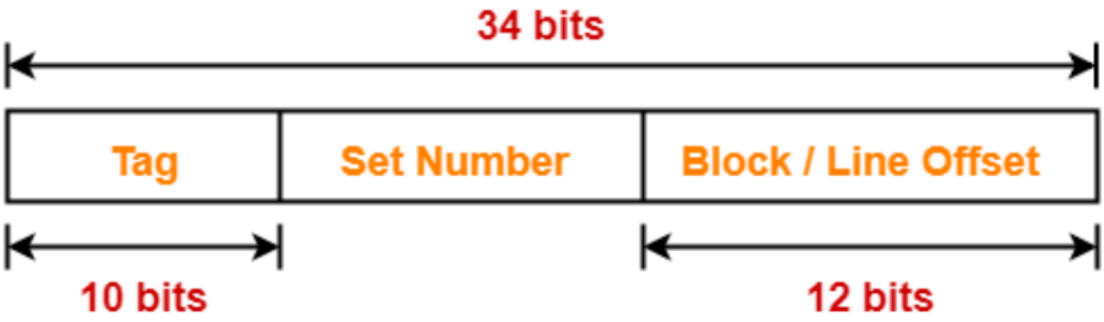
We have,

Block size

= 4 KB

= 2^{12} bytes

Thus, Number of bits in block offset = 12 bits



Number of Bits in Set Number-

Number of bits in set number

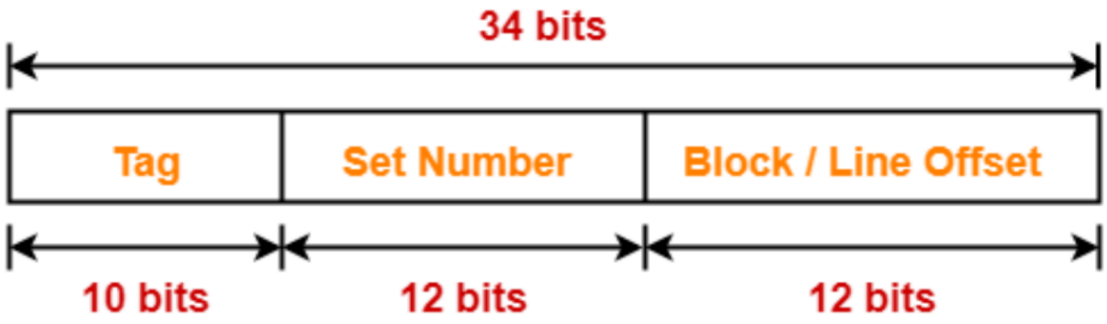
= Number of bits in physical address – (Number of bits in tag + Number of bits in block offset)

= 34 bits – (10 bits + 12 bits)

= 34 bits – 22 bits

= 12 bits

Thus, Number of bits in set number = 12 bits



Number of Sets in Cache-

We have-

Number of bits in set number = 12 bits

Thus, Total number of sets in cache = 2^{12} sets

Number of Lines in Cache-

We have-

Total number of sets in cache = 2^{12} sets

Each set contains 4 lines

Thus,

Total number of lines in cache

= Total number of sets in cache x Number of lines in each set

= 2^{12} x 4 lines

= 2^{14} lines

Size of Cache Memory-

Size of cache memory

= Total number of lines in cache x Line size

= 2^{14} x 4 KB

$= 2^{16}$ KB

= 64 MB

Thus, Size of cache memory = 64 MB

Tag Directory Size-

Tag directory size

= Number of tags x Tag size

= Number of lines in cache x Number of bits in tag

$= 2^{14}$ x 10 bits

= 163840 bits

= 20480 bytes

= 20 KB

Thus, size of tag directory = 20 KB

Problem-04:

Consider a 8-way set associative mapped cache. The size of cache memory is 512 KB and there are 10 bits in the tag. Find the size of main memory.

Solution-

Given-

- Set size = 8
- Cache memory size = 512 KB
- Number of bits in tag = 10 bits

We consider that the memory is byte addressable.

Let-

- Number of bits in set number field = x bits
- Number of bits in block offset field = y bits

Sum of Number Of Bits Of Set Number Field And Block Offset Field-

We have,

Cache memory size = Number of sets in cache x Number of lines in one set x Line size

Now, substituting the values, we get-

512 KB = 2^x x 8 x 2^y bytes

2^{19} bytes = 2^{3+x+y} bytes

19 = 3 +x + y

x + y = 19 – 3

x + y = 16

Number of Bits in Physical Address-

Number of bits in physical address

= Number of bits in tag + Number of bits in set number + Number of bits in block offset

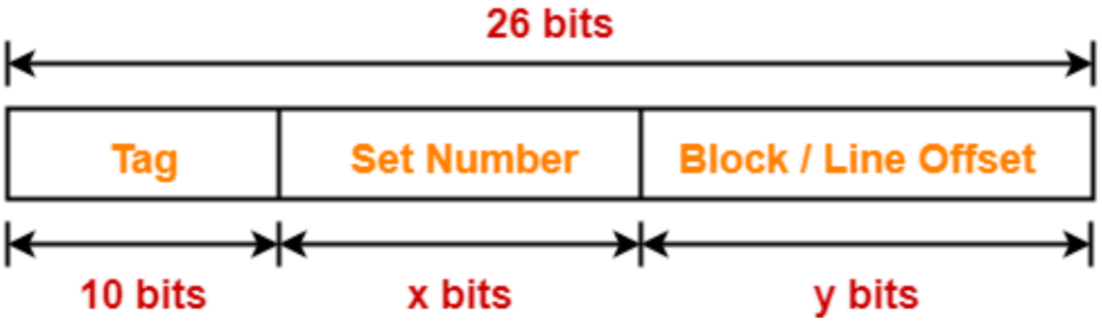
= 10 bits + x bits + y bits

= 10 bits + (x + y) bits

= 10 bits + 16 bits

= 26 bits

Thus, Number of bits in physical address = 26 bits



Size of Main Memory-

We have,

Number of bits in physical address = 26 bits

Thus, Size of main memory

= 2^{26} bytes

= 64 MB

Thus, size of main memory = 64 MB

Problem-05:

Consider a 4-way set associative mapped cache. The size of main memory is 64 MB and there are 10 bits in the tag. Find the size of cache memory.

Solution-

Given-

- Set size = 4
- Main memory size = 64 MB
- Number of bits in tag = 10 bits

We consider that the memory is byte addressable.

Number of Bits in Physical Address-

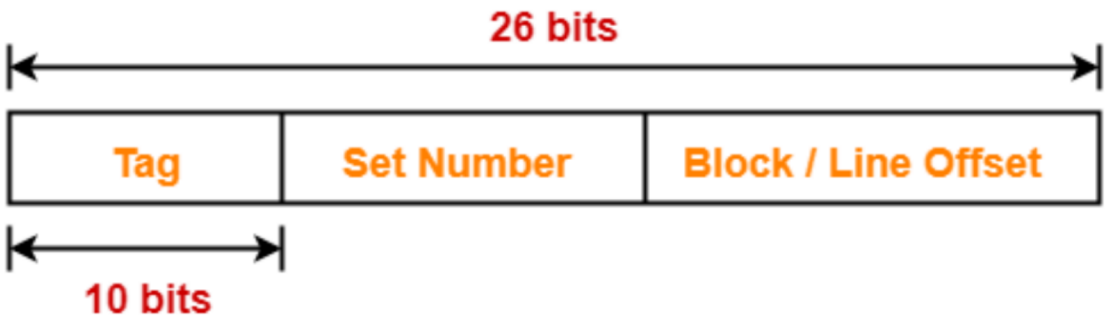
We have,

Size of main memory

= 64 MB

= 2^{26} bytes

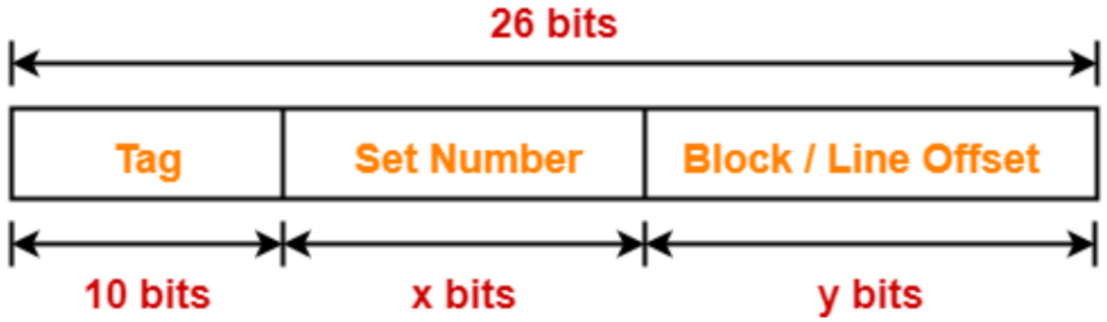
Thus, Number of bits in physical address = 26 bits



Sum Of Number Of Bits Of Set Number Field And Block Offset Field-

Let-

- Number of bits in set number field = x bits
- Number of bits in block offset field = y bits



Then, Number of bits in physical address

= Number of bits in tag + Number of bits in set number + Number of bits in block offset

So, we have-

26 bits = 10 bits + x bits + y bits

$26 = 10 + (x + y)$

$x + y = 26 - 10$

$x + y = 16$

Thus, Sum of number of bits of set number field and block offset field = 16 bits

Size of Cache Memory-

Cache memory size

= Number of sets in cache x Number of lines in one set x Line size

= 2^x x 4 x 2^y bytes

= 2^{2+x+y} bytes

= 2^{2+16} bytes

= 2^{18} bytes

= 256 KB

Thus, size of cache memory = 256 KB